LOCAL ELECTION

Context of local election is real, the numbers expressing the number of votes and percentage are not as „nice" in the real world as they are in our exercises (percentage is mostly a rounded number, therefore it is impossible to calculate exactly the number of votes, we deal with this problem in the section Presidential elections or in the Exercise 5 in the section The rate of unemployment). In this case our aim was not to practise inaccurate (rounded) numbers and so we decided to choose „nice " numbers in order to concentrate on different aims.

The theme consists of two parts:

The first part consists of the exercises 1 to 5. First two are focused on reading comprehension, i.e. they test how much the pupils really understand the given diagrams. Exercises 3,4,5 test the pupils´ ability to work with percentage.

Exercise 6 is to make the pupils´guessing easier in the exercises 7 and 8 (by choosing suitable numbers). This exercise can be solved by a teacher together with his pupils or the teacher can show the pupils how to solve this problem. The important thing is that the pupils understand that the numbers they are supposed to suggest in exercises 7 and 8 have to be the multiples of 20 (and each one of them has to be bigger than 50).

With the exercise 7 we recommend to work in groups or with a whole class. The work should be followed by a discussion about possible approaches in dealing with this problem.

Exercise 8 should be solved by the pupils individually (on the basis of the discussion from the exercise 7), it is possible to give this task to the pupils after a lapse from the exercise 7.

In the Horný koniec district.

1. **30 %**
2. **80 votes**
3. By **20 votes**. There are two possible ways:
   - We find out the number of votes obtained by Eva (40% from 200 is 80) and by Stanislav (30% from 200 is 60) and then we subtract,
   - we subtract the gained percentage of Eva and Stanislav (40% - 30%= 10%) and we figure out 10% from 200

   It is useful to try both ways of calculation with pupils.
4. Number of votes obtained by each candidate is:

   Stanislav: 40 + 5 + 60 = 105

   Rudolf: 30 + 10 + 30 = 70

   Karol: 20 + 15 + 30 = 65

   Eva: 10 + 20 + 80 = 110

   Eva would win the election.
5. $35\,\% = \frac{35}{100} = \frac{7}{20}$, $30\,\% = \frac{30}{100} = \frac{3}{10}$. The amounts of votes have to be the whole numbers, so the ballot in Dolný koniec region has to be such a number P that $\frac{7}{20}$ and also $\frac{3}{10}$ from it – it means $\frac{7}{20}P$ and $\frac{3}{10}P$ - are whole numbers. Therefore P has to be a multiple of 20.

   (From the same thought we can derive the fact that for the Centrum region it would be the multiple of 5, for the Horný koniec region the multiple of 20. As the numbers of votes for Centrum region is $800 - P_{Hk} - P_{Dk}$ where the number 800 as well as the numbers of votes P_{Hk} , P_{Dk} in the Horný koniec region and Dolný koniec region are the multiples of 20 then the number of votes for the Centrum region also has to be the multiple of 20.)

We assume that the pupils will try to solve the task by guessing and probably it will not be totally an unreasonable try. They might take into consideration that

- Karol defeated Stanislav only in the Dolný koniec region and therefore we cannot choose a small number for the total number of votes in this region,
- in the Centrum region Karol obtained smaller percentage of the votes than Eva and also Stanislav did, therefore we have to put there few votes of the total number of 800 votes.

Karol will win if the distribution of the votes is: 100, 600, 100 (in that case the total numbers of obtained votes would be: Stanislav 256, Karol 275, and Eva 260).

Another possibility is an algebraic approach: If we mark the numbers of votes in the regions *A, B, C* then Stanislav gets $0{,}4A + 0{,}3B + 0{,}45C$ of the votes, Karol gets $0{,}2A + 0{,}35B + 0{,}45C$ of the votes and Eva gets $0{,}4A + 0{,}35B + 0{,}1C$ of the votes. According to the assignment in the exercise:

$$0{,}2A + 0{,}35B + 0{,}45C > 0{,}4A + 0{,}3B + 0{,}45C, \quad \text{hence} \quad B > 4A. \qquad (*)$$

and at the same time

$$0{,}2A + 0{,}35B + 0{,}45C > 0{,}4A + 0{,}35B + 0{,}1C, \quad \text{hence} \quad 7C > 4A. \qquad (**)$$

Suitable numbers *A, B, C* fulfilling the inequality (*) and also (**) we can find by experimenting, e.g.: We can choose A= 60 (that is the smallest multiple of 20 bigger than 50, have a look at the text before the Exercise 6 and the solution of the Exercise 7). Then from (*) and from (**) we get $B > 240$, $7C > 240$. Eventually we choose *B, C* in such a way that these inequalities are fulfilled, the sum $B + C$ is 740 and B, C are the multiples of 20 (look at the solution of the Exercise 7). Such *B, C* are e.g. $B = 280$, $C = 460$.

Conditions (*), (**) can be used when you check the pupils´ solutions: the correct ones of the Exercise 6 are only those which meet the conditions (*), (**) in addition to $A + B + C = 800$, $A, B, C \geq 50$ and *A, B, C* are the multiples of 20.)

(Eva would win e.g. if the distribution of the votes were 300, 440, 60 (obtained amounts of the votes would be Stanislav 279, Karol 241 and Eva 280). While guessing we have to take into consideration that in the Horný koniec region she suffered from a loss, therefore it is important to put there only few votes from the ballot (800 votes). The smallest possible number is 60 (it has to be a number bigger than 50 and at the same time a multiple of 20).

By repeating the same algebraic approach from the Exercise 7 we get these conditions

$$0{,}4A + 0{,}35B + 0{,}1C > 0{,}4A + 0{,}3B + 0{,}45C, \quad \text{hence} \quad B > 7C, \qquad (+)$$

$$0{,}4A + 0{,}35B + 0{,}1C > 0{,}2A + 0{,}35B + 0{,}45C, \quad \text{hence} \quad 4A > 7C. \qquad (++)$$

Therefore only those solutions are correct which fulfill these conditions $B > 7C$, $4A > 7C$, $A + B + C = 800$, $A, B, C \geq 50$ and *A, B, C* are the multiples of 20. These conditions are fulfilled only for C = 60, where the number A is one of the numbers 120, 140, ..., 300 and $B = 740 - A$.

(For C = 80 the solution doesn´t exist: If we chose C = 80, the smallest multiples of 20 which fulfilled the conditions (+) and (++) would be the numbers $B = 580$, $A = 160$. But for these numbers the sum $A + B + C$ is bigger than 800.)